Proc. of the 18th International Modal Analysis Conference, IMAC; San Antonio, TX; 2000

COMPUTATIONAL MODEL UPDATING OF LARGE SCALE FINITE ELEMENT MODELS

Carsten Schedlinski

ICS – Dr.-Ing. Carsten Schedlinski
P.O. box 1648, 63206 Langen, Germany

www.ics-solutions.de

ABSTRACT

In this paper a model validation strategy is presented that integrates computational model updating techniques. A special in-house software package is utilized that makes use of a wide range of the analysis capabilities of a commercial Finite Element solver, especially the eigenvalue and the sensitivity module. The software package thus allows for efficient computational model updating of large scale Finite Element models.

At first the basic theory is summarized. Then the model validation strategy is explained. Finally an application to an automotive transmission will underline the effectiveness of the procedure.

NOMENCLATURE

0	linearization point
A	'initial'
dof	degree of freedom
EMA	experimental modal analysis
FEA	Finite Element analysis
i, j	indices
J	objective function
T	'test'
G	sensitivity matrix
H(jω)	frequency response vector/matrix
K, **M**	stiffness/mass matrices
p	design parameter vector
W, $\mathbf{W}_p$	weighting matrices
x	mode shape vector
z	residual vector
α, β	design parameters
ω	eigenvalue vector
ξ	modal damping degree vector

1 INTRODUCTION

To validate Finite Element models, test data, e.g. from an experimental modal analysis, may be utilized. If the deviations between test and analysis are not acceptable, the idealization of the investigated elastomechanical system must be reviewed. If the structure of the Finite Element model with respect to discretization, chosen element types etc. is correct, the test/analysis deviations can be minimized by modification of appropriate physical model parameters (shell thicknesses, Young's moduli, mass densities etc.). If only single parameters are considered a 'manual' update based on engineering skills may be successful. However, proceeding this way bears limits for real elastomechanical systems due to the large number of parameters to be considered. Here computational model updating techniques must be applied which allow for a simultaneous update of multiple model parameters.

In this paper a model validation strategy is presented that integrates computational model updating techniques. A special MATALB® based software package is utilized, that has originated at the University of Kassel, Lightweight Structures and Structural Mechanics Laboratory, Germany and is now maintained in cooperation with ICS. The software makes use of a wide range of the analysis capabilities of MSC.Nastran™, especially the eigenvalue solver and the sensitivity module. The software package thus allows for efficient computational model updating of *large scale* Finite Element models.

At first the basic theory is summarized. Then the model validation strategy is explained. Finally an application to an automotive transmission will underline the effectiveness of the procedure.

2 MODEL UPDATING THEORY

2.1 Updating of physical parameters

The basis for computational model updating of physical stiffness and mass parameters is the parameterization of the model matrices according to equations (1) (see references [4], [5]). This parameterization allows for *local* updating of uncertain model areas.

$$\mathbf{K} = \mathbf{K}_A + \sum \alpha_i \mathbf{K}_i \quad , \quad i = 1...n_\alpha \tag{1a}$$
$$\mathbf{M} = \mathbf{M}_A + \sum \beta_j \mathbf{M}_j \quad , \quad j = 1...n_\beta \tag{1b}$$

with:

$\mathbf{K}_A$, $\mathbf{M}_A$ initial analytical stiffness/mass matrices
$\mathbf{p} = [\alpha_i\ \beta_j]$ vector of unknown design parameters
$\mathbf{K}_i$, $\mathbf{M}_j$ given substructure matrices defining location and type of model uncertainties

Using equations (1) and appropriate residuals (containing different test/analysis differences) the following objective function can be derived:

$$J(\mathbf{p}) = \Delta\mathbf{z}^T\ \mathbf{W}\ \Delta\mathbf{z} + \mathbf{p}^T\ \mathbf{W}_p\ \mathbf{p} \rightarrow \min \tag{2}$$

with: $\Delta\mathbf{z}$ residual vector
$\mathbf{W}$, $\mathbf{W}_p$ weighting matrices

The minimization of equation (2) yields the desired design parameters **p** while the second term is used to constrain the parameter variation. The weighting matrix $\mathbf{W}_p$ has to be selected with care since for $\mathbf{W}_p >> \mathbf{0}$ no design parameter changes will occur.

The residuals $\Delta\mathbf{z} = \mathbf{z}_T - \mathbf{z}(\mathbf{p})$ ($\mathbf{z}_T$: test data vector, $\mathbf{z}(\mathbf{p})$: corresponding analytical data vector) are usually nonlinear functions of the design parameters. Thus the minimization problem is also nonlinear and must be solved iteratively. One way is to apply the classical sensitivity approach (see reference [2]) where the analytical data vector is linearized at point 0 by a Taylor series expansion truncated after the first term. Proceeding this way leads to:

$$\Delta\mathbf{z} = \Delta\mathbf{z}_0 - \mathbf{G}_0\ \Delta\mathbf{p} \tag{3}$$

with:

$\Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0$ design parameter changes
$\Delta\mathbf{z}_0 = \mathbf{z}_T - \mathbf{z}(\mathbf{p}_0)$ test/analysis difference at linearization point 0
$\mathbf{G}_0 = \partial\mathbf{z}/\partial\mathbf{p}|_{p=p_0}$ sensitivity matrix at linearization point 0
$\mathbf{p}_0$ design parameter vector at linearization point 0

If the design parameters are not bounded the minimization problem (2) leads to the linear problem (4) which has to be solved in each iteration step for the actual linearization point.

$$(\mathbf{G}_0^T \mathbf{W}\ \mathbf{G}_0 + \mathbf{W}_p)\ \Delta\mathbf{p} = \mathbf{G}_0^T \mathbf{W}\ \Delta\mathbf{z}_0 \tag{4}$$

For $\mathbf{W}_p = \mathbf{0}$ equation (4) represents a standard weighted least squares problem. Of course any other mathematical minimization technique may be applied as well to solve equation (2).

Two important residuals are natural frequency and mode shape residuals. The residual vector in this case takes the form:

$$\Delta\mathbf{z}_0 = \begin{bmatrix} \omega_T - \omega \\ \mathbf{x}_T - \mathbf{x} \end{bmatrix}_0 \tag{5}$$

with:

ω_T, ω test/analysis vectors of natural frequencies
$\mathbf{x}_T$, $\mathbf{x}$ test/analysis mode shape vectors

The sensitivity matrix for the residual vector introduced in equation (5) is given in equation (6).

$$\mathbf{G}_0 = \begin{bmatrix} \partial\omega / \partial\mathbf{p} \\ \partial\mathbf{x} / \partial\mathbf{p} \end{bmatrix}_0 \tag{6}$$

For the calculation of the derivatives please refer to references [4], [5].

2.2 Updating of modal parameters

In order to update modal damping parameters the classic sensitivity approach can also be applied. The residual in this case is:

$$\Delta\mathbf{z}_0 = \left[\mathbf{H}_T(j\omega) - \mathbf{H}(j\omega)\right]_0 \tag{7}$$

with: $\mathbf{H}_T(j\omega)$ measured frequency response
$\mathbf{H}(j\omega)$ analytical frequency response

Partial differentiation of the analytical frequency response with respect to the modal damping degrees yields the sensitivity matrix:

$$\mathbf{G}_0 = \left[\frac{\partial\mathbf{H}(j\omega)}{\partial\xi}\right]_0 \tag{8}$$

with: ξ vector of modal damping degrees

The calculation of the sensitivity matrix according to (8) is very simple if proportional damping is assumed. A more detailed introduction can be found in reference [4].

3 MODEL VALIDATION

Computational model updating techniques can be utilized in order to validate FE models. By updating multiple selected parameters of the model simultaneously a minimization of the test/analysis differences can be achieved.

The model is validated in two steps:

1. updating of stiffness and mass properties (physical parameters)
2. updating of modal damping (modal parameters)

In a first step only stiffness and mass properties (physical parameters) are updated by minimizing the test/analysis differences e. g. of eigenvalues and mode shapes. A special MATLAB® program is utilized here that takes advantage of the MSC.Nastran™ analysis capabilities (in particular the sensitivity module). Thus large scale FE models can be processed. The parameter changes are directly applied to the ‘Bulk Data’ section (the section in which the FE model is defined) of the MSC.Nastran™ input deck. Proceeding this way allows for an update of any physical parameter that can be defined in MSC.Nastran™

(e. g. shell thicknesses, beam section properties, Young's moduli, mass densities).

In order to handle complex elastomechanical systems a decomposition into components is usually necessary (reduction of uncertain parameters). The components are individually updated and the quality of the (modified) assembly is assessed subsequently. If the model quality is not yet sufficient further updating may be performed considering only the interface parameters (e. g. stiffnesses of connection elements).

A central problem when utilizing computational model updating is to select the design parameters. Next to engineering judgement computational localization methods may be applied (see e. g. [3]).

Another possibility for parameter selection is to determine the most sensitive design parameters by performing a sensitivity study. Here the sensitivity matrix according to equation (6) is calculated for multiple design parameters. A subsequent investigation of the sensitivity matrix can help to detect parameters which allow for a significant change of the analytical model behavior. However, the sensitivity study does not allow to assess the physical relevance of a design parameter. It merely reflects a design parameter's potential to change the considered behavior of the model.

If dynamic analyses are to be conducted, modal damping (modal parameters) may be updated in a second step. Here the differences between measured and analytic frequency responses are minimized in the vicinity of the resonance peaks. This is accomplished utilizing another special MATLAB® program. The overall goal is to achieve high quality FE analysis predictions (at least in the covered frequency range) and thus a validated model.

An appropriate experimental data basis is very important for the success of subsequent computational model updating. Thus thorough test design must be performed in advance.

When using experimental modal analysis data, test design should cover the following aspects:

- selection of target modes
- selection of measurement dof w. r. t.:
  - coincidence of measurement and FE dof
  - sufficient spatial resolution of the target modes
- selection of exciter locations
- definition of frequency resolution

Selection of target modes:

In order to obtain sufficient information w. r. t. the global stiffness of the model all global elastic modes in the considered frequency range should be selected as target modes. I. e. these modes are to be excited and observed in the test. In addition, sufficient spatial resolution of all elastic modes in the considered frequency range should be provided in order to avoid spatial aliasing in any case.

Selection of measurement dof:

Selection of measurement dof is performed in three steps:

1. principal assessment of required measurement information
2. selection of measurement dof based on coincidence with FE nodes and accessibility
3. check of validity of selected measurement dof

A principal assessment of necessary measurement information is made utilizing a special in-house pickup-selection software which is based on a maximum linear independence criterion of the mode shapes. With the knowledge from the principal assessment the final selection is made and checked by calculating the Auto-MAC of the target modes at the selected measurement dof (see reference [6]).

Selection of exciter locations:

In order to determine the optimal exciter locations, frequency responses are calculated for the selected measurement dof. Candidate exciter locations are pre-selected w. r. t. their ability to excite the target modes using an in-house exciter-selection software (see reference [6]). For each candidate exciter location univariate mode indicator functions are calculated according to [1]. Based on these mode indicator functions the exciter locations are chosen such that every target mode is sufficiently excited by at least one of the exciters.

Frequency resolution:

If the modal damping of the first elastic modes can be estimated, a minimum frequency spacing can be derived e. g. from response calculations. This is mandatory in order to acquire enough data for the subsequent experimental modal analysis algorithms.

4 EXAMPLE: AUTOMOTIVE TRANSMISSION

The procedure is demonstrated by way of an automotive transmission housing, Fig. 1. Goal is to properly predict the dynamics of the housing in the frequency range from zero to 2000 Hz. The updated model shall afterwards be utilized to investigate alternative modeling strategies of the gear set.

The complete model consists of 19800 nodes and 16833 elements (shell and volume elements). It is assembled from a front (12925 nodes and 11008 elements) and a rear housing (6670 nodes and 5742 elements). The bolt connection of the two components is idealized with beam elements.

4.1 Experimental Modal Analysis

For the two components and for the assembly individual free/free vibration tests with subsequent experimental modal analyses were performed. Data were acquired from zero to 3000 Hz. However, above 2000 Hz the confidence in the identified modal data is rather low since due to high modal density and increasing damping influence EMA inherent mode separation problems occurred. A possible poor correlation above 2000 Hz is thus not necessarily caused by a lack in FE model quality - it may also result from a more or less erroneous experimental data base (especially the mode shapes).

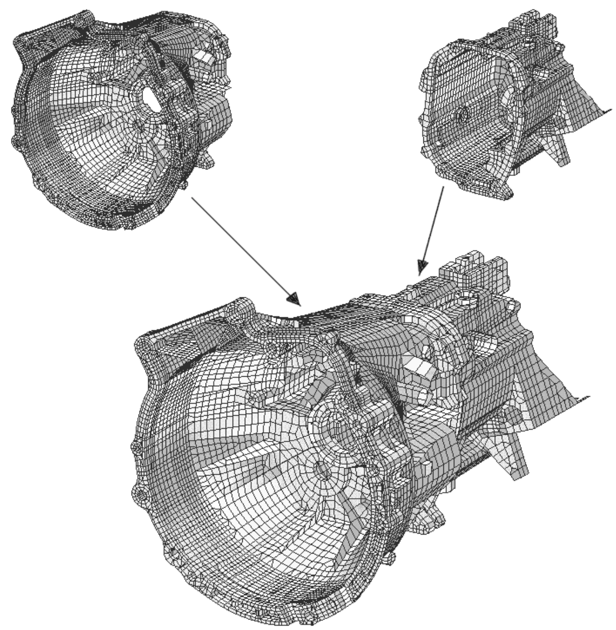

Fig. 1: FE models, components and assembly (BMW, Munich, Germany)

4.2 Initial correlation

The correlation of test and analysis data is checked via the MAC value. The results are presented in tables 1 to 3. Fig. 2 shows a visualization of the MAC matrix for the assembly.

Table 1: Front housing – initial correlation

FEA # [1)]	EMA #	Frequency [Hz] FEA	Frequency [Hz] EMA	Dev. [%]	MAC [%] (> 60 %)
1	1	381.04	388.01	-1.80	96.10
2	2	411.25	417.01	-1.38	94.02
3	3	856.00	874.49	-2.11	96.61
4	4	955.91	995.68	-3.99	97.96
5	5	1461.37	1501.41	-2.67	95.28
7	6	1732.01	1717.42	0.85	81.58
6	7	1690.71	1841.45	-8.19	61.21
8	8	1896.28	2023.81	-6.30	75.32

[1)] without rigid body modes.

Table 2: Rear housing - initial correlation

FEA # [1)]	EMA #	Frequency [Hz] FEA	Frequency [Hz] EMA	Dev. [%]	MAC [%] (> 60 %)
1	1	495.87	520.52	-4.74	98.54
2	2	562.51	578.63	-2.79	98.72
3	3	1152.05	1163.75	-1.01	96.82
4	4	1336.25	1356.05	-1.46	94.05
5	5	1664.34	1769.70	-5.95	96.81
9	9	2338.46	2371.01	-1.37	82.63

[1)] without rigid body modes.

Table 3: Assembly - initial correlation

FEA # [1)]	EMA #	Frequency [Hz] FEA	Frequency [Hz] EMA	Dev. [%]	MAC [%] (> 60 %)
1	1	468.86	505.87	-7.32	89.47
2	2	494.31	531.61	-7.02	82.27
3	3	862.95	886.39	-2.64	97.56
4	4	959.60	1002.81	-4.31	94.68
5	5	1036.44	1225.14	-15.40	72.27
6	6	1220.77	1342.71	-9.08	79.63
8	8	1361.99	1439.23	-5.37	70.40
9	10	1418.46	1603.33	-11.53	78.30
12	12	1734.00	1740.89	-0.40	75.76
11	13	1616.47	1845.28	-12.40	67.31
13	14	1760.52	1941.11	-9.30	65.08
14	15	1910.21	2041.99	-6.45	75.00
21	19	2298.01	2382.90	-3.56	66.24

[1)] without rigid body modes.

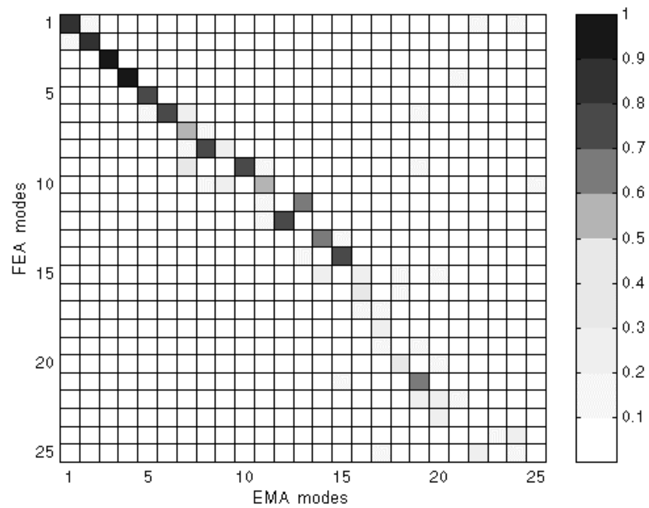

Fig. 2: Assembly – initial MAC matrix

In the frequency range of interest the correlation is already quite good for the components. For the assembly the level of MAC values is relatively low above 1000 Hz. In addition larger frequency deviations can be found. Especially FEA mode 5 (global bending about vertical transmission axis) deviates about 15 % from the EMA result.

4.3 Updating of physical parameters

4.3.1 Components

For the components multiple updating runs were performed using different design parameter sets. The best results could be produced by updating only Young's moduli of selected FE model areas (the updating of Young's moduli bears the advantage of not altering the overall mass of the FE model). It was found that some parameters changed rather excessively. This is an indicator that the parameters have lost their physical significance. The resulting model thus plays the role of a substitute model that fulfills the requirement of reproducing the test data. Since - for this application - a proper modeling of the housing dynamics was mandatory the changes were retained and accepted.

A typical selection of design parameters can be found in Fig. 3; a representative updating run is presented in Fig. 4. It can be observed that the mean frequency deviation tends towards zero and that the standard deviation is reduced. This is typical for computational model updating.

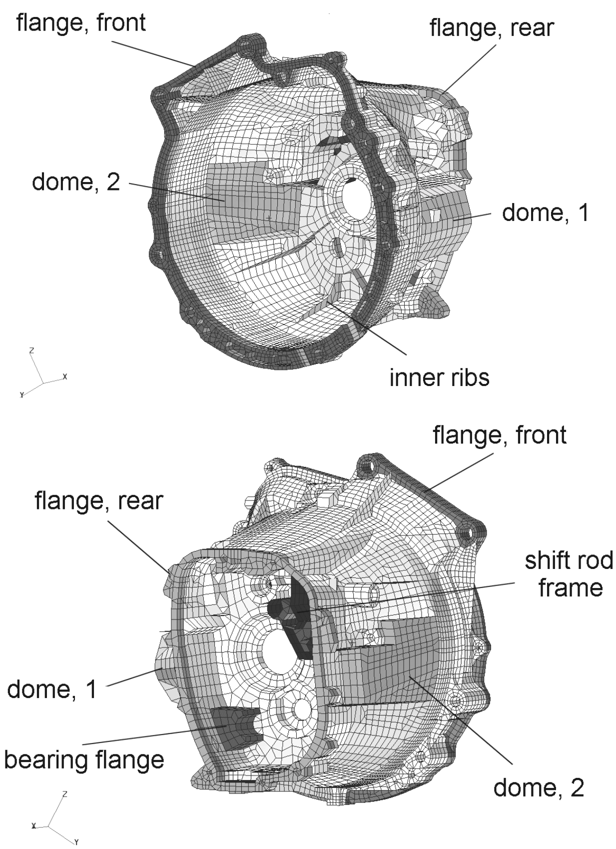

Fig. 3: Front housing - design parameters (Young's moduli)

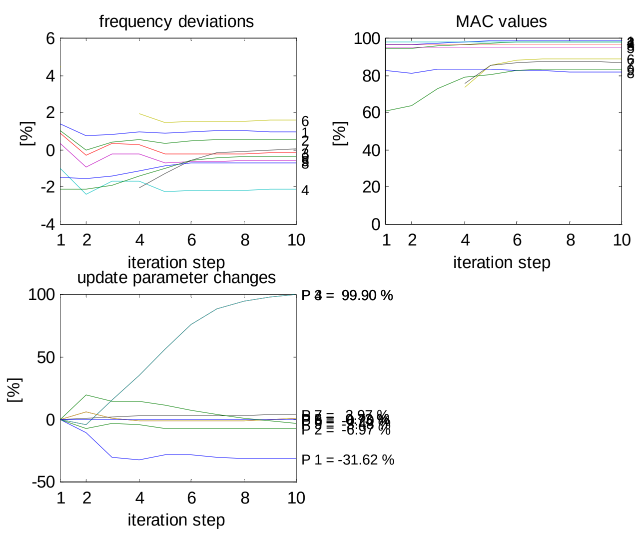

Fig. 4: Front housing - typical updating results

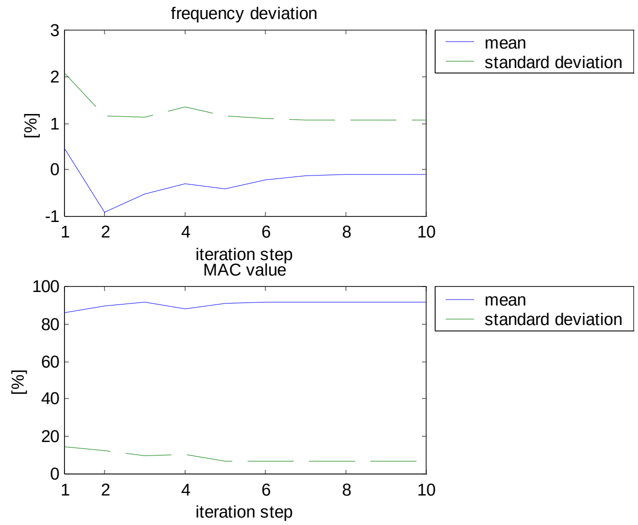

Fig. 4 (cont.): Front housing - typical updating results

4.3.2 Assembly

For the assembly multiple updating runs were performed as well. The best results could be produced by updating the sections (A) and the area moments of inertia (I1/I2) of the connecting beam elements.

The design parameter changes can be found in table 4. Especially the extensive change of the area moments of inertia is to be noted here. The resulting design parameters again play the role of substitute parameters which are required to model the real connectivity stiffness.

Table 4: Assembly – design parameter changes

Param.	Change [%]	Param.	Change [%]	Param.	Change [%]
A_1	80.41	A_6	-98.82	A_{11}	255.67
A_2	83.44	A_7	28.04	$I1_{1\text{-}11}$	275774.03
A_3	113.40	A_8	43.41	$I2_{1\text{-}11}$	105513.54
A_4	397.78	A_9	28.04		
A_5	-52.48	A_{10}	28.04		

The correlation after updating is found in table 5; the corresponding MAC matrix is presented in Fig. 5. FEA modes 1 through 14 and 17 can be paired while the MAC values are mostly higher than 80 %. Except for modes 11, 13 and 14 the frequency deviations are smaller than 3 %. Mode 5 (global bending about vertical transmission axis) can now be paired with a frequency deviation of less than 3 % (over 15 % before updating) and a MAC value of over 90 %. EMA mode 9 cannot be paired to analytical results. However, this mode was ranked less trustworthy during the modal extraction process and can be neglected. All in all a very good correlation in the frequency range up to 2000 Hz could be achieved.

Beyond 2000 Hz practically no correlation can be found. However, because of the low confidence in the test data base in this frequency range, no definitive assessment of the FE model quality can be made here anymore.

Table 5: Assembly – correlation after updating

FEA # [1)]	EMA #	Frequency [Hz] FEA	Frequency [Hz] EMA	Dev. [%]	MAC [%] (> 60 %)
1	1	505.16	505.87	-0.14	88.64
2	2	527.44	531.61	-0.78	81.25
3	3	883.14	886.39	-0.37	97.74
4	4	986.44	1002.81	-1.63	95.06
5	5	1194.50	1225.14	-2.50	90.36
6	6	1309.20	1342.71	-2.50	92.19
7	7	1389.28	1415.86	-1.88	86.72
8	8	1422.50	1439.23	-1.16	84.07
9	10	1556.22	1603.33	-2.94	86.75
10	11	1651.55	1695.16	-2.57	75.83
12	12	1772.60	1740.89	1.82	80.21
11	13	1740.83	1845.28	-5.66	67.91
13	14	1857.76	1941.11	-4.29	74.58
14	15	1976.44	2041.99	-3.21	65.59
17	17	2224.15	2258.11	-1.50	64.38

[1)] without rigid body modes.

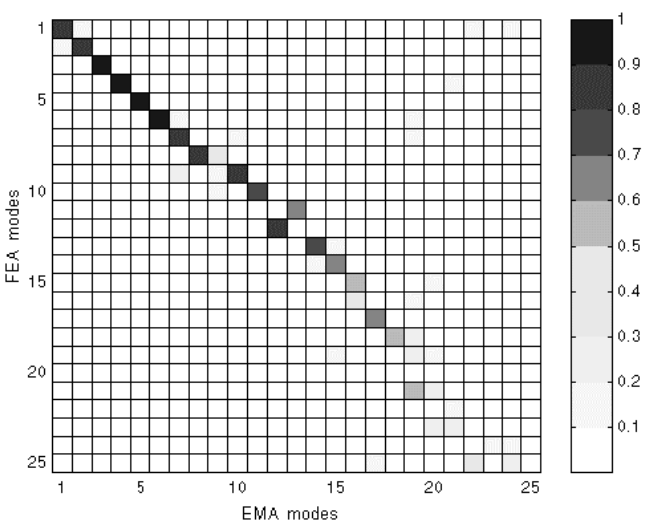

Fig. 5: Assembly – MAC matrix after updating

4.4 Updating of modal parameters

Updating of modal damping degrees of the assembly is performed in a last step. The identified EMA damping values serve as starting values here. Since the damping update is based on a minimization of response deviations at the resonances, only the damping degrees of the 15 paired modes may be updated. The results are listed in table 6.

Fig. 6 shows the measured and calculated mean value of response amplitudes before and after updating (radial excitation at the front flange of the assembly). The updating of physical parameters significantly improves the response behavior. Even above 2000 Hz an improvement can be noted which increases the confidence in the updated model. The updating of modal damping degrees reduces the deviations even more. Here the overall amplitude level of measured and calculated responses is adjusted. All together the model is validated up to about 2000 Hz.

Table 6: Results of damping update

EMA #	1	2	3	4	5
EMA damp. [%]	0.23	0.32	0.27	0.29	0.25
updated damp. [%]	0.34	0.28	0.31	0.34	0.63

EMA #	6	7	8	9	10
EMA damp. [%]	0.31	0.34	0.55	0.66	0.34
updated damp. [%]	0.35	0.22	0.55	0.32	1.07

EMA #	11	12	13	14	17
EMA damp. [%]	0.44	0.58	0.63	0.36	0.92
updated damp. [%]	1.15	0.58	0.63	0.85	1.03

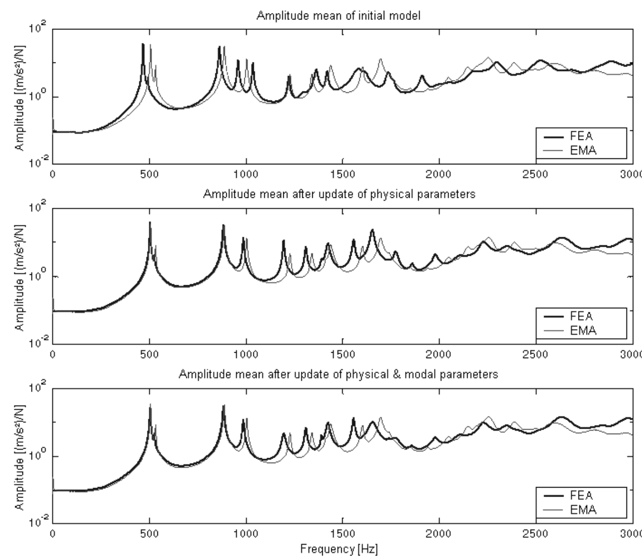

Fig. 6: Frequency responses before / after updating

5 CONCLUSIONS

By way of an automotive transmission housing the effectiveness of a model validation strategy that integrates computational model updating techniques is demonstrated.

For the transmission housing, which is to be utilized to investigate alternative modeling strategies of the gear set, a very good correlation of test and analysis data could be achieved in the frequency range from zero to about 2000 Hz. The model can therefore be considered validated in this frequency range. Even above 2000 Hz an improvement of the frequency response can be noted, which increases the confidence in the updated model.

6 ACKNOWLEDGEMENTS

The author wishes to thank Mr. Ingo Seeber of BMW, Munich, Germany for the good cooperation throughout the transmission updating project and BMW, Munich, Germany for the approval to present the results.

7 REFERENCES

[1] **Breitbach, E.**
Neuere Entwicklungen auf dem Gebiet des Standschwingungsversuchs an Luft- und Raumfahrt-strukturen,
VDI-Report No. 221, Germany, 1974

[2] **Collins, J. D. et al.**
Statistical Identification Of Structures
AIAA J., Vol. 12, No. 2, pp. 185-190, 1974

[3] **Lallement, G.**
Localisation Techniques
Proceedings of Workshop 'Structural Safety Evaluation Based on System Identification Approaches', Vieweg, Braunschweig/Wiesbaden, Germany, 1988

[4] **Link, M.**
Updating of Analytical Models - Review of Numerical Procedures and Application Aspects
Structural Dynamics Forum SD 2000, Los Alamos, USA, April 1999

[5] **Natke, H. G.**
Einfuehrung in die Theorie und Praxis der Zeitreihen- und Modalanalyse
3., ueberarb. Aufl., Vieweg, Braunschweig/ Wies-baden, Germany, 1992

[6] **Schedlinski, C. and Link, M.**
An Approach to Optimal Pick-Up and Exciter Placement
Proceedings of the 14th IMAC, Dearborn, Michigan, USA, 1996